5/14/18 "Too many of us are not living our dreams because we are living our fears."-Les Brown

HW: "Systems of Linear Equations" Homework #1-2
Test 2 on Wednesday 5/23

AIM: How do we solve a system of equations?

Warm Up:

Solve the following systems of equations algebraically by either substitution or elimination:

solve for one variable and plug that in to the other equation

combine the equations in a way to get rid of one variable

1. $2x - y = -1$
$2x + y = -7$

$4x = -8$

$\frac{4x}{4} = \frac{-8}{4}$

$x = -2$

Solution

$\boxed{(-2, -3)}$

Need to find y:

use $2x - y = -1$

$2(-2) - y = -1$

$-4 - y = -1$

$+4 \qquad +4$

$-y = 3$

$\frac{-y}{-1} = \frac{3}{-1}$

$y = -3$

OR

$2x + y = -7$

$2(-2) + y = -7$

$-4 + y = -7$

$+4 \qquad +4$

$y = -3$

$$\begin{array}{rrcl} & \cancel{2x} - y & = & -1 \\ - & \cancel{2x} + y & = & -7 \\ \hline & -2y & = & 6 \end{array}$$

$$\frac{-2y}{-2} = \frac{6}{-2}$$

$$y = -3$$

2. $2x + 2y = 3$
$x = 4y - 1$

Substitution

$2(4y-1) + 2y = 3$
$8y - 2 + 2y = 3$
$+2 \qquad\qquad +2$
$\frac{10y}{10} = \frac{5}{10}$
$y = \frac{1}{2}$

Find x:
$x = 4y - 1$
$x = 4\left(\frac{1}{2}\right) - 1$
$x = 2 - 1$
$x = 1$

$\boxed{\left(1, \frac{1}{2}\right)}$

3. $2(x - 2y = 3)$
$-2x + 4y = 1$

$\rightarrow 2x - 4y = 6$
$-2x + 4y = 1$
$0x + 0y = 7$

$0 = 7$ Does not make sense

therefore $\boxed{\text{no Solution}}$

4. $2(2x - y = 1) \rightarrow 4x - 2y = 2$
$4x - 2y = 2 \rightarrow 4x - 2y = 2$
$0x + 0y = 0$

$0 = 0 \leftarrow$ Does make sense

Infinite Solutions

5. $5(3x + 2y = 2) \rightarrow 15x + 10y = 10$
$-3(5x + 7y = -4) \rightarrow -15x - 21y = 12$

$\frac{-11y}{-11} = \frac{22}{-11}$
$y = -2$

To find x:
$3x + 2y = 2$
$3x + 2(-2) = 2$
$3x - 4 = 2$
$3x = 6$
$x = 2$

$\boxed{(2, -2)}$

6. $3x + 2y = -9 \rightarrow 3x + 2y = -9$
$2(2x + y = -7) \rightarrow 4x + 2y = -14$

$\frac{-1x}{-1} = \frac{5}{-1}$
$x = -5$

To find y:
$3x + 2y = -9$
$3(-5) + 2y = -9$
$-15 + 2y = -9$
$2y = 6$
$y = 3$

$\boxed{(-5, 3)}$

Summary:

1) One unique solution (One point)

2) No solution (Parallel Lines) * inconsistent system

3) Infinite Solutions (Equations are of the same line) * Dependent System

HW check:

1) $x + y = 5$
$x - y = 39$

$2x = 44$

$\frac{2x}{2} = \frac{44}{2}$

$x = 22$

Find y:

$x + y = 5$

$22 + y = 5$

$-22 \quad -22$

$y = -17$

$$\boxed{x = 22,\ y = -17 \text{ OR } (22, -17)}$$

2) $4x + 3y = -13$

$y = 6x - 8$

$4x + 3(6x - 8) = -13$

$4x + 18x - 24 = -13$

$22x - 24 = -13$

$+24 \quad +24$

$22x = 11$

$\frac{22x}{22} = \frac{11}{22}$

$x = \frac{1}{2}$

Find y:

$y = 6x - 8$

$y = 6\left(\frac{1}{2}\right) - 8$

$y = 3 - 8$

$y = -5$

$$\boxed{x = \frac{1}{2},\ y = -5 \text{ OR } \left(\frac{1}{2}, -5\right)}$$

You should be very familiar with solving two-by-two systems of linear equations (two equations and two unknowns). In this lesson, we will extend the method of **elimination** to linear systems of three equations and three unknowns. These linear systems serve as the basis for a field of math known as **Linear Algebra**.

7. Consider the three-by-three system of linear equations shown below. Each equation is numbered in this first exercise to help keep track of our manipulations.

(1) $2x+y+z=15$
(2) $6x-3y-z=35$
(3) $-4x+4y-z=-14$

(a) The **addition property of equality** allows us to add two equations together to produce a third valid equation. Create a system by adding equations (1) and (2) and (1) and (3). Why is this an effective strategy in this case?

we used all equations and eliminated z.

$$\begin{aligned} (1)\quad & 2x+y+\cancel{z}=15 \\ +\ (2)\quad & 6x-3y-\cancel{z}=35 \\ \hline & 8x-2y=50 \quad (4) \end{aligned}$$

$$\begin{aligned} (1)\quad & 2x+y+\cancel{z}=15 \\ +\ (3)\quad & -4x+4y-\cancel{z}=-14 \\ \hline & -2x+5y=1 \quad (5) \end{aligned}$$

(b) Use this new two-by-two system to solve the three-by-three.

$8x-2y=50 \longrightarrow \cancel{8x}-2y=50$

$4(-2x+5y=1) \longrightarrow \oplus\ -\cancel{8x}+20y=4$

$$\begin{aligned} 18y &= 54 \\ \tfrac{18y}{18} &= \tfrac{54}{18} \\ y &= 3 \end{aligned}$$

Find x: (use either Eq(4) or (5))

$-2x+5y=1$
$-2x+5(3)=1$
$-2x+15=1$
$-15\ \ -15$
$\frac{-2x}{-2}=\frac{-14}{-2}$
$x=7$

Find z: (Use either Eq(1), (2) or (3))

$2x+y+z=15$
$2(7)+(3)+z=15$
$14+3+z=15$
$17+z=15$
$-17\ \ -17$
$z=-2$

$$\boxed{\begin{aligned} x&=7 \\ y&=3 \quad \text{OR } (7,3,-2) \\ z&=-2 \end{aligned}}$$

Just as with two by two systems, sometimes three-by-three systems need to be manipulated by the **multiplication property of equality** before we can eliminate any variables.

8. Consider the system of equations shown below. Answer the following questions based on the system.

$$\text{①}\quad 4x + y - 3z = -6$$
$$\text{②}\quad -2x + 4y + 2z = 38$$
$$\text{③}\quad 5x - y - 7z = -19$$

(a) Which variable will be easiest to eliminate? Why? Use the multiplicative property of equality and elimination to reduce this system to a two-by-two system.

y b/c one equation has +y another has −y so adding will eliminate

① $4x + y - 3z = -6$
$+$
③ $5x - y - 7z = -19$

④ $9x - 10z = -25$

✱ Now we need to use ② and have to eliminate y

① $-4(4x + y - 3z = -6) \rightarrow -16x - 4y + 12z = 24$

$+$

② $-2x + 4y + 2z = 38 \rightarrow -2x + 4y + 2z = 38$

⑤ $-18x + 14z = 62$

(b) Solve the two-by-two system from (a) and find the final solution to the three-by-three system.

④ $2(9x - 10z = -25) \rightarrow 18x - 20z = -50$

⑤ $-18x + 14z = 62 \rightarrow -18x + 14z = 62$

$+$

$$-6z = 12$$
$$\frac{-6z}{-6} = \frac{12}{-6}$$
$$\boxed{z = -2}$$

solve for x:

$9x - 10(-2) = -25$

$9x + 20 = -25$

$\quad -20 \quad -20$

$\frac{9x}{9} = \frac{-45}{9}$

$x = -5$

Solve for y:

$4(-5) + y - 3(-2) = -6$

$-20 + y + 6 = -6$

$-14 + y = -6$

$+14 \quad +14$

$y = 8$

$$\boxed{\begin{aligned} x &= -5 \\ y &= 8 \\ z &= -2 \end{aligned}}$$

(X) Make sure everything is lined up before you begin

ex:

$$3x+2y=7$$
$$3y-2=x$$
$$\Rightarrow$$
$$3x+2y=7$$
$$-x+3y=2$$

9. Solve the system of equations shown below. Show each step in your solution process.

$$4x - 2y + 3z = 23$$
$$x + 5y - 3z = -37$$
$$-2x + y + 4z = 27$$